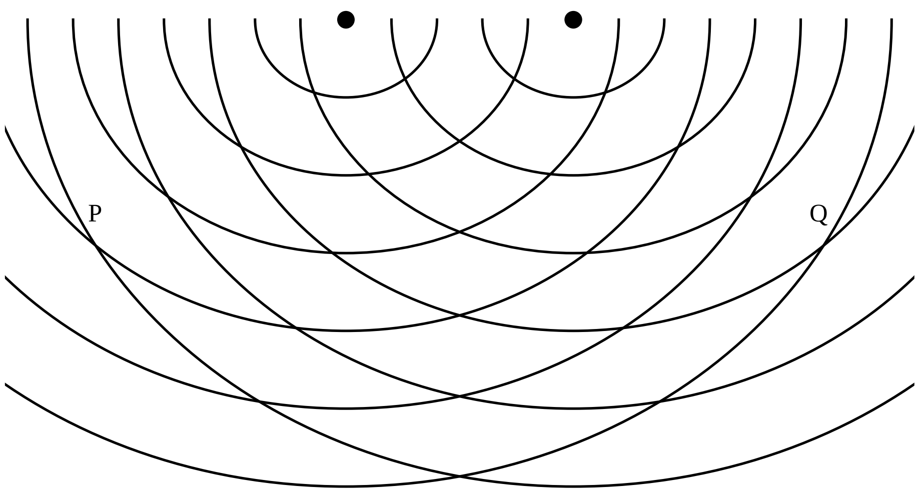
Teacher notes
Topic C

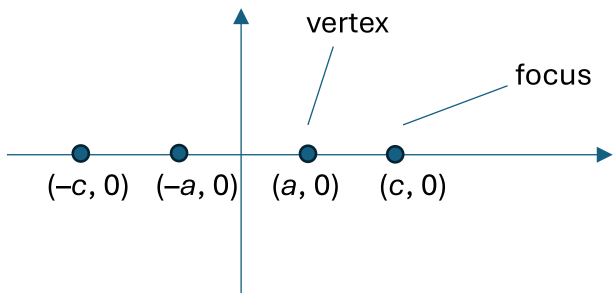
A detailed explanation of problem 14.24 in the textbook.

The figure shows wavefronts emitted from two identical sources. An observer walks along the straight line from P to Q.



On the axes, draw a sketch graph to show how the intensity of sound heard by the observer varies with distance from P.

The first thing to do is to draw the curves that give the same path difference. If a point is at distances d_1 and d_2 from the sources then the condition for maxima is $|d_1 - d_2| = n\lambda$. This is the defining equation of a hyperbola with the sources being at the foci of the hyperbola. The diagram shows the foci and the vertices of the hyperbola.



The equation of the hyperbola is $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ where $c^2 = a^2 + b^2$.

In fact, $|d_1 - d_2| = 2a$ and so $2a = n\lambda$. This means that $b^2 = c^2 - \dfrac{n^2\lambda^2}{4}$.

So we have the equation of the hyperbola:

$$y = \pm b\sqrt{\frac{x^2}{a^2} - 1}$$

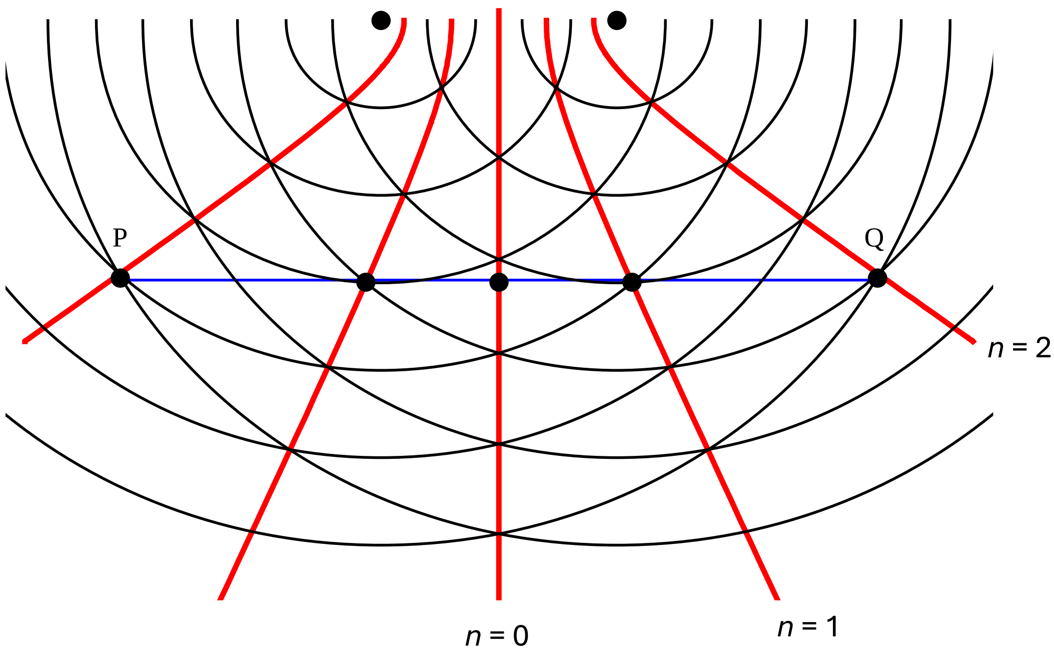
$$y = \pm\sqrt{c^2 - \frac{n^2\lambda^2}{4}}\sqrt{\frac{4x^2}{n^2\lambda^2} - 1}$$

We got this by using hyperbola properties. We could also derive this by using $|d_1 - d_2| = n\lambda$ i.e.

$$\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = n\lambda$$

but this would take a page of algebra.

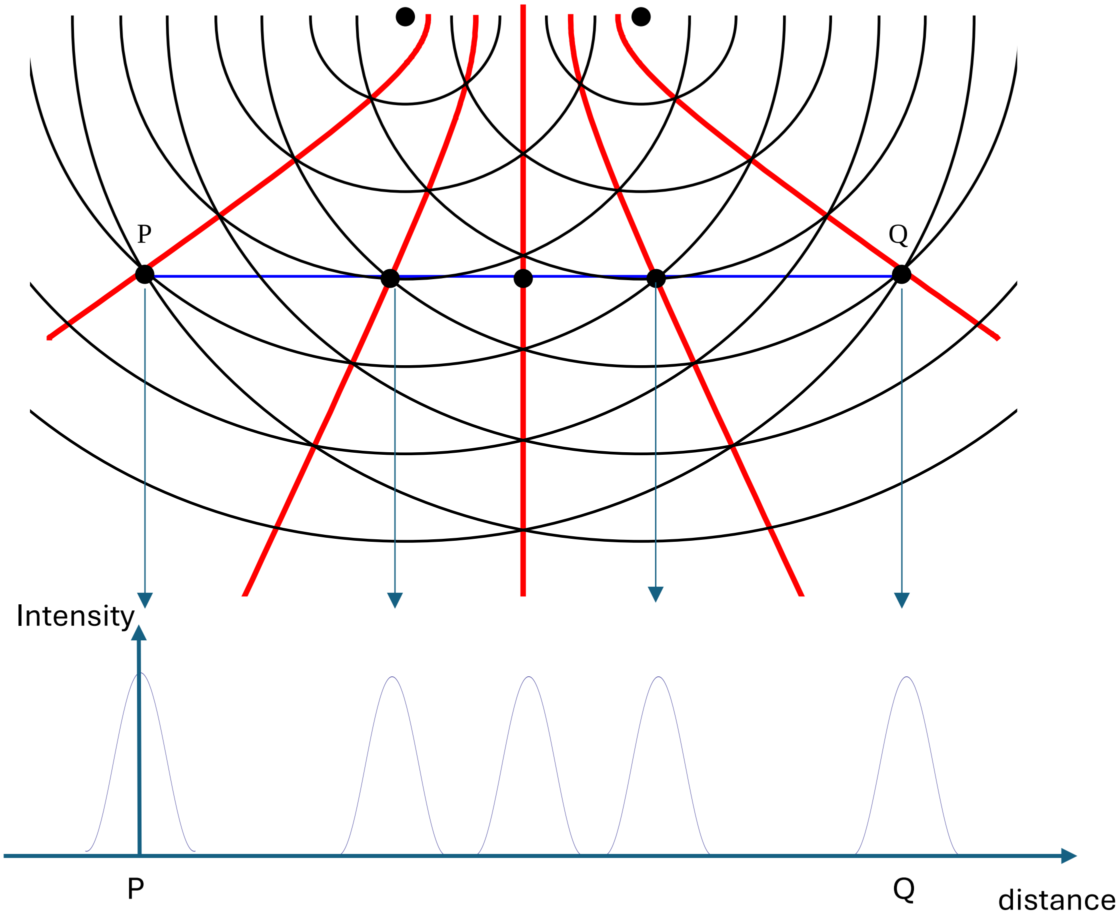
We now plot the hyperbolas for $n = 0, \pm 1, \pm 2$. We use $c = 0.25$ m and $\lambda = 0.20$ m. The results are:



The hyperbolas intersect the blue line joining P and Q at the points shown by dots. These are points of maximum intensity.

Obviously, we did not need all the machinery about hyperbolas to draw the hyperbolas. We could join points at the intersection of wavefronts with a smooth curve. But it is always nice to be precise.

Hence the graph giving the intensity as a function of the distance from P along the blue line is given by:



The maxima are not uniformly separated. This is because we are too close to the sources. We drew all maxima to have the same intensity. In reality this cannot be the case since the distances from the sources are different so the intensity must be dropping as we move away from the central maximum in either direction.

To see what happens **far** from the sources we use the asymptotes for the hyperbola. For the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ the asymptotes are $y=\pm\frac{b}{a}x$. So, if we imagine a "screen" a distance D from the sources, the asymptotes intersect the line $y=-D$ at points given by $D=\mp\frac{b}{a}x$ i.e. $x=\mp\frac{Da}{b}$.

Now, in a typical experiment with light D is very large and $b^2 = c^2 - \dfrac{n^2\lambda^2}{4} \approx c^2$ since also $c >> \lambda$. Hence $x \approx \mp \dfrac{Da}{c} = \mp \dfrac{Dn\lambda}{2c}$.

But $2c = d$, the separation of the sources or the separation of the slits if you like. Hence $x \approx \mp \dfrac{Dn\lambda}{d}$.

The separation of points of maximum intensity along the screen is then

$$s = \Delta x = \frac{D(n+1)\lambda}{d} - \frac{Dn\lambda}{d} = \frac{D\lambda}{d}.$$

This is independent of n and so the maxima are equally separated, and we derived the booklet formula for the separation in a completely different way!